Journal of Mathematical Chemistry, Vol. 39, No. 2, February 2006 (© 2005)
DOI: 10.1007/s10910-005-9031-3

A discussion on the Einstein–Podolski–Rosen (EPR) effect* in a unique wavefunction quantum mechanical framework

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Received 7 June 2005; revised 11 August 2005

A unique wavefunction, constructed according to the original Einstein–Podolski–Rosen (EPR) description, is used here to analyze the behavior of two equal non-interacting quantum systems: S(1) and S(2).The results show that the EPR paradox, which will be referred as EPR *effect* in this work, always appears in this theoretical context. When the expectation value of a Hermitian operator is sought, it can yield two different values, when measured on S(1) or S(2). However, the EPR effect disappears if the wavefunction is chosen symmetric or antisymmetric by interchanging the S(1) and S(2) coordinates. The same EPR effect appears when the expectation value of a state selector projection operator is computed on S(1) or S(2), but it disappears within a symmetric or antisymmetric EPR wavefunction form. On the other hand, the action of the selectors over the EPR wavefunction provide images on EPR system S(1) or S(2) which could be different, so the EPR effect persists except if, similarly as in the statistical case, a wavefunction constructed with a symmetric or antisymmetric superposition with respect the EPR system coordinates is used. Thus, with appropriate wavefunction choices, in statistical expectation value measures and non-statistical selector images as well, the EPR effect could not persist.

KEY WORDS: Einstein–Podolski–Rosen paradox, wavefunction structure, expectation values, inward matrix product, selectors

1. Introduction

In a recent work [1] the so-called Einstein–Podolski–Rosen (EPR) paradox [2], which will be hereafter called the *EPR effect*, was analyzed from the

*This second paper on EPR paradox is also expressly dedicated to the memory Professor Einstein, in order to celebrate the 100th anniversary of the first paper on relativity theory, but also to commemorate the 75 anniversary of the publication of EPR paradox, thus, as before, Professors Podolski and Rosen are also included in the homage.

point of view of the classical quantum mechanical theory of statistical measures, while admitting the EPR framework based on non-interacting states of two equal systems and two non-commuting operators. The result was such that the EPR effect was not present in this statistical quantum measurement way. The present discussion follows a different pathway using a unique wavefunction instead of employing the two equivalent functions of the original EPR formulation. The present analysis shows that, when studying the statistical expectation values as in the first study, the EPR effect can be overridden by appropriate symmetry choices of the EPR wavefunction.

The present work starts taking into account a similar framework as previously admitted, but keeping the mathematical development within a unique operator and one wavefunction only. It will be shown that one can arrive to some stage where the EPR effect appears. Then, the results of the present paper will illustrate how to override the EPR effect explicitly, taking into account the symmetry or antisymmetry of the wave function coefficients, when choosing a statistical expectation value frame. All the same, it will be studied how is not difficult to overcome the EPR effect too when measuring a single state projection operator, a selector, by means of the same quantum mechanical expectation value technique and using the unique EPR wavefunction. However, when choosing the selector as a non-statistical projector over the unique EPR wavefunction, the EPR effect becomes persistent as in the original EPR formulation and other manipulations shall be devised to get through the EPR effect.

2. The starting points

First, this discussion will take into account the following basic points in order to set up the EPR framework [2]:

(1) Two equal submicroscopic systems S(1) and S(2) have interacted in the past and now are in a non-interacting stationary state. One can refer to these systems as *EPR systems*.

(2) Some Hermitian operator A, which can act on both EPR systems, is known.

(3) The secular equation of the operator A is known and can be written for both EPR systems as:

$$Aa_k(i) = \alpha_k a_k(i); (i = 1, 2)\,, \tag{1}$$

where:

$$\{\alpha_k\} \wedge \{a_k\,(i)\}\,; (i = 1, 2) \tag{2}$$

are the eigenvalues and eigenfunctions for the operator A. By the symbol $(i)\,;(i=1,2)$ there are represented the variables describing both EPR systems.

Therefore, the eigenfunctions of the operator A form an orthonormalized set in the following sense:

$$\left\langle a_k \mid a_l \right\rangle = \int_D a_k^* (1)\ a_l (1)\, \mathrm{d}V_1 = \int_D a_k^* (2)\ a_l (2)\, \mathrm{d}V_2 = \delta_{kl}. \tag{3}$$

(4) A known complete set of functions, which can have the role of coefficients in a EPR wavefunction development, can be related to EPR systems S(1) and S(2) and written as:

$$G = \{g_k (1)\} \wedge F = \{f_k (2)\}, \tag{4}$$

where the symbols (1) and (2) refer to the variables of the EPR systems. One can name this set (4) as the *EPR coefficient function set*.

3. The EPR wavefunction

One can easily write the EPR wavefunction for the joint non-interacting systems, just using the information of the four previous points as stated in the previous paragraph. The EPR wavefunction will be referred to the eigenfunctions of system S(1) as a basis set, with the EPR coefficient functions of S(2) of equation (4) acting as coordinates as well:

$$\Psi (1,2) = \sum_k f_k (2)\ a_k (1). \tag{5}$$

Taking into account that the role of both systems coordinates could be exchanged without any further difficulty.

Now, as the eigenfunctions of the operator A are to be considered a complete set too, then this property can be used to express the EPR coefficient function set F in the usual way, with the eigenfunctions of the operator A acting in turn as a basis set and some specific coordinates for each EPR coefficient function:

$$\forall k\colon\ f_k (2) = \sum_l c_{lk} a_l (2), \tag{6}$$

therefore, the EPR wavefunction (5) can be expressed as:

$$\Psi (1,2) = \sum_k \left[\sum_l c_{lk} a_l (2) \right] a_k (1) = \sum_k \sum_l c_{lk} a_l (2)\, a_k (1). \tag{7}$$

Collecting the EPR coefficients into a matrix: $\mathrm{C} = \{c_{lk}\}$, which will be called hereafter the *EPR coefficient matrix*, and ordering the eigenfunctions in a vector structure like:

$$\mathbf{a}(i) = \{a_p(i)\}\ (i = 1, 2), \tag{8}$$

then, the EPR wavefunction expression above can be written, using the complete sum[1] of the inward matrix product (IMP)[2] of two factors (3), composed by the EPR coefficient matrix and the tensor product of the vectors (8), as:

$$\Psi(1, 2) = \langle \mathrm{C} * (\mathbf{a}(2) \otimes \mathbf{a}(1)) \rangle .$$

4. Normalization of EPR wavefunction

Normalization of the EPR wavefunction (7) provides a first insight into the nature of the EPR coefficient matrix **C**. The Euclidian norm of the EPR function (7) has the following form:

$$\langle \Psi \mid \Psi \rangle = \sum_{k,l} \sum_{p,q} c_{lk}^{*} c_{qp} \langle a_l \mid a_q \rangle \langle a_k \mid a_p \rangle ,$$

which, taking into account the orthonormalization relationships (3), transforms into:

$$\langle \Psi \mid \Psi \rangle = \sum_{k,l} \sum_{p,q} c_{lk}^{*} c_{qp} \delta_{lq} \delta_{kp} = \sum_{k,l} |c_{lk}|^2 = 1.$$

Thus, the set of squared modules of the EPR coefficients **C** correspond to a discrete probability distribution, which constitutes a well-known property in quantum mechanical wavefunction description [4, 5]. This property of the EPR coefficients is nicely written in terms of the complete sum of the IMP of the EPR coefficient matrix and its complex conjugate: $\mathbf{C}^*$:

$$\mathbf{D} = \mathbf{C}^* * \mathbf{C} = \{d_{lk} = |c_{lk}|^2\} \rightarrow \langle \mathbf{D} \rangle = 1. \tag{9}$$

Such a matrix manipulation transforms the EPR coefficient matrix into a positive definite valued one, an element of a semispace unit shell [6, 7].

5. Expectation values of the EPR wavefunction under the operator A

Having set the trivial relationship involved in the EPR wavefunction normalization, in a first instance, the expectation value of the operator A acting

[1] The complete sum of a matrix $\mathbf{A} = \{a_{ij}\}$ is defined as: $\langle \mathbf{A} \rangle = \sum_{i,j} a_{ij}$

[2] The inward matrix product (IMP) of two matrices $\{\mathbf{A}, \mathbf{B}\}$ is defined as: $\mathbf{P} = \mathbf{A} * \mathbf{B} \rightarrow \forall i, j : p_{ij} = a_{ij} b_{ij}$.

over the system S(1) can be written as:

$$\begin{aligned}\langle\Psi|\,A\,(1)\,|\Psi\rangle &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\left\langle a_l\middle|a_q\right\rangle \langle a_k|\,A\left|a_p\right\rangle \\ &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\delta_{lq}\alpha_p\left\langle a_k\middle|a_p\right\rangle \\ &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\delta_{lq}\alpha_p\delta_{kp} \\ &= \sum_{k,l} c_{lk}^{*}c_{lk}\alpha_k = \sum_{k,l}|c_{lk}|^2\,\alpha_k.\end{aligned} \tag{10}$$

Now, collecting the eigenvalue spectrum of the operator A into a diagonal matrix:

$$\mathrm{A} = \mathrm{Diag}\,(\alpha_k)\,, \tag{11}$$

then, the expectation value of the same operator can be written as a trace of the matrix product:

$$\langle A\,(1)\rangle = Tr\left(\mathbf{CAC}^{+}\right), \tag{12}$$

where C^{+} is the conjugate-transpose of the EPR coefficient matrix, or using the previous definition of the diagonal matrix (11), the matrix (9) and the complete sum of their IMP one arrives at the equivalent description:

$$\langle A\,(1)\rangle = \langle \mathbf{D} * \mathbf{A}\rangle\,.$$

On the other hand, the application of the operator A over the system S(2), produces:

$$\begin{aligned}\langle\Psi|\,A\,(2)\,|\Psi\rangle &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\,\langle a_l|\,A\left|a_q\right\rangle\left\langle a_k\middle|a_p\right\rangle \\ &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\delta_{kp}\alpha_q\left\langle a_l\middle|a_q\right\rangle \\ &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\delta_{kp}\alpha_q\delta_{lq} \\ &= \sum_{k,l} c_{lk}^{*}c_{lk}\alpha_l = \sum_{k,l}|c_{lk}|^2\,\alpha_l\end{aligned}$$

a slightly different result from the previous one, as it can be seen when the corresponding trace matrix expression is employed:

$$\langle A\,(2)\rangle = Tr\left(\mathbf{C}^{+}\mathbf{AC}\right). \tag{13}$$

Expression (13) can be also expressed in terms of the matrix (9), like in the previous case:

$$\langle A\,(1)\rangle = \langle \mathbf{A} * \mathbf{D}\rangle\,.$$

Thus, as in general there is no reason forbidding that expressions (12) and (13) provide each one a different result, a new form of the EPR effect appears in this case, when comparing both expectation values.

Apparently, then, when using the normalized wave function (7), *a different result could be obtained* when the measure of the same observable, represented by the Hermitian operator A, is performed in system S(1) or S(2).

It seems, therefore, that the EPR objection, which was originally based on an observable *single measure* wavefunction collapse [4] in a two wavefunction representation [2] framework, can be extended to a *statistical measure* of the observable within a unique wavefunction representation.

An analysis of this situation follows, in order to obtain information on under which circumstances the encountered EPR effect will disappear or persist.

6. Conditions for solving the EPR effect using the equality

$$\left\langle A\,(1)\right\rangle = \left\langle A\,(2)\right\rangle.$$

At first instance one can try to see how the EPR effect can be surmounted in the case of the preceding paragraph, as it has been already performed in the original EPR framework statistical case, as discussed in the preceding paper [1].

The equality between expressions (12) and (13), can be sought by using the equality:

$$\sum_{k,l} |c_{lk}|^2 \alpha_k = \sum_{k,l} |c_{lk}|^2 \alpha_l \rightarrow \sum_{k,l} |c_{lk}|^2 (\alpha_l - \alpha_k) = 0. \tag{14}$$

The precedent result permit that the following situations may be worth of study:

(1) The equality (14) will hold trivially in the case when the spectrum of the operator A is completely degenerate. Thus, the diagonal matrix containing its eigenvalues is a scalar matrix:

$$\mathbf{A} = \alpha \mathbf{I} \rightarrow \forall k, l : \alpha_k = \alpha_l = \alpha.$$

This scenario is hardly interesting, as an objective EPR system experimental situation, having such a characteristic it is not so common in practice, as far as one can imagine when dealing with atoms and molecules. Obviously, in this situation, the EPR coefficient matrix can have arbitrary elements.

(2) When, on the contrary than in the situation above, the spectrum of the Hermitian operator is completely non-degenerate, one can write the

equality (14), as:

$$\sum_k \sum_{l<k} |c_{lk}|^2 (\alpha_l - \alpha_k) = -\sum_k \sum_{l>k} |c_{lk}|^2 (\alpha_l - \alpha_k) \\ = \sum_k \sum_{l>k} |c_{lk}|^2 (\alpha_k - \alpha_l) = \sum_k \sum_{l<k} |c_{kl}|^2 (\alpha_l - \alpha_k),$$

which implies:

$$\sum_k \sum_{l<k} \left(|c_{lk}|^2 - |c_{kl}|^2\right) (\alpha_l - \alpha_k) = 0$$

and therefore, the following equalities will hold:

$$\forall k > l \colon |c_{lk}|^2 = |c_{kl}|^2 . \tag{15}$$

On the other hand, the relationships (15) constitute a set of conditions, usually fulfilled by both Hermitian and Skewhermitian matrices. For instance, Hermitian matrices fulfill:

$$\forall k, l \colon c_{lk} = c_{kl}^* \rightarrow |c_{lk}|^2 = c_{lk}^* c_{lk} = c_{kl} c_{lk} = c_{kl} c_{kl}^* = |c_{kl}|^2$$

and in the same manner, Skewhermitian matrices fulfill:

$$\forall k, l : c_{lk} = -c_{kl}^* \rightarrow |c_{lk}|^2 = c_{lk}^* c_{lk} = (-c_{kl})\, c_{lk} \\ = (-c_{kl}) \left(-c_{kl}^*\right) = c_{kl} c_{kl}^* = |c_{kl}|^2 .$$

Thus, one can obtain the equality of both expectation values when the spectrum of the Hermitian operator is completely non-degenerate and the EPR coefficient matrix is Hermitian or Skewhermitian.
Each one of the conditions above implies, in turn, that the EPR wave function has to be symmetric or antisymmetric with respect exchange of the EPR systems coordinates:

$$\Psi(1, 2) = \pm \Psi(2, 1)$$

a result which coincides with the usual quantum mechanical description of bosons and fermions, respectively.

(3) When the spectrum of the operator A appears possessing some degenerate states, the situation it is not so clear as in the previous points and one shall study this situation separately. Perhaps, one can provisionally say that using the EPR coefficients arbitrariness within the degenerate part of the spectrum, then, one can consider taking a coherent Hermitian or Skewhermitian coefficient set as in the non-degenerate part.

Then, this resulting situation appears to be similar to the one encountered in the previous work. The EPR effect disappears within the scenario of a statistical measure of a quantum mechanical expectation value, when boson or fermion symmetry properties of the EPR wavefunction are taken into account.

7. The EPR wavefunction collapsed state measurement and the persistence of the EPR effect

7.1. Selectors

The Hermitian operator A, when one takes into account its spectral decomposition in terms of the eigenvalues and eigenfunctions, can have the form:

$$A = \sum_s \alpha_s \left|a_s\right\rangle \left\langle a_s\right| = \sum_s A_s ,$$

moreover, the set of Hermitian operators $\{A_s\}$ can be interpreted as the ones which select, by projection of the EPR wavefunction, a given system state. One can name them *selectors*.

The selectors A_s act over the EPR wavefunction selecting the sth state measure of the operator A, when applied to system S(1):

$$\begin{aligned} A_s(1)[\Psi] &= \sum_{k,l} c_{lk} A_s(1)[a_k(1)]\, a_l(2) \\ &= \sum_{k,l} c_{lk} \alpha_s a_s(1)\, \delta_{sk} a_l(2) \\ &= \left(\sum_l c_{ls} a_l(2)\right) \alpha_s a_s(1) \\ &= \alpha_s f_s(2)\, a_s(1) \end{aligned} \tag{16}$$

and when applied to system S(2):

$$\begin{aligned} A_s(2)[\Psi] &= \sum_{k,l} c_{lk} a_k(1)\, A_s(2)[a_l(2)] \\ &= \sum_{k,l} c_{lk} a_k(1)\, \alpha_s a_s(2) \delta_{sl} \\ &= \left(\sum_k c_{sk} a_k(1)\right) \alpha_s a_s(2) \\ &= \alpha_s g_s(1)\, a_s(2) \end{aligned} \tag{17}$$

by using the next expression to define similar EPR functions of type (4) for system S(1), in the manner of equation (6) for system S(2):

$$g_s(1) = \sum_k c_{sk} a_k(1) \,. \tag{18}$$

The only way to assure that the EPR effect will not be present in this case is to assume that for every selector:

$$\forall s:\ f_s(x) = g_s(x)\,, \tag{19}$$

or what is the same:

$$\forall s \wedge k: c_{ks} = c_{sk}. \tag{20}$$

Such a property implies that the EPR coefficient matrix shall be symmetric. This situation seems different from the previous result when the expectation values of the Hermitian operator were studied, but resembles the EPR result for the case when selectors of two Hermitian non-commuting operators are applied over the EPR wavefunction represented in the basis of each operator [2].

Conditions (19) or (20) constitute a quite particular constraint, which has not always to be fulfilled. In this sense *the EPR effect persists*, when selector images of the EPR wavefunction over each one of the EPR systems are obtained, unless the EPR coefficient matrix becomes a symmetrical one. This will be true whenever such a symmetric EPR coefficient matrix can be employed to construct the EPR wavefunction.

One can also try to obtain information about this case from the Gramian of the EPR coefficient functions. For such a purpose, the norms and scalar products of two EPR coefficient functions shall be evaluated, that is:

$$\begin{aligned}\forall s: \langle f_s \mid f_s\rangle &= \sum_k \sum_l c_{ks}^* c_{ls} \langle a_k \mid a_l\rangle \\ &= \sum_k c_{ks}^* c_{ks} = \sum_k |c_{ks}|^2 = \left[\mathbf{C}^+\mathbf{C}\right]_{ss}\end{aligned} \tag{21}$$

with also:

$$\begin{aligned}\forall s: \langle g_s \mid g_s\rangle &= \sum_k \sum_l c_{sk}^* c_{sl} \langle a_k \mid a_l\rangle \\ &= \sum_k c_{sk}^* c_{sk} = \sum_k |c_{sk}|^2 = \left[\mathbf{C}\mathbf{C}^+\right]_{ss}\end{aligned} \tag{22}$$

and provided the scalar product of both functions is performed like:

$$\begin{aligned}\forall s: \langle f_s \mid g_s\rangle &= \sum_k \sum_l c_{ks}^* c_{sl} \langle a_k \mid a_l\rangle = \sum_k \sum_l c_{ks}^* c_{sl} \delta_{kl} \\ &= \sum_k c_{ks}^* c_{sk} = \left[\mathrm{CC}^*\right]_{ss}.\end{aligned}$$

Then, the Gramian of the two functions can be, thus, written:

$$\forall s: \Gamma\left(f_s; g_s\right) = \left(\sum_k |c_{ks}|^2\right)\left(\sum_k |c_{sk}|^2\right) - \left(\sum_k c_{ks}^* c_{sk}\right)^2$$

this is the same as:

$$\forall s:\Gamma\left(f_s;g_s\right)=\left(\sum_k\sum_l c_{ks}^{*}c_{ks}c_{sl}^{*}c_{sl}\right)-\left(\sum_k\sum_l c_{ks}^{*}c_{sk}c_{ls}^{*}c_{sl}\right)$$
$$=\sum_k\sum_l c_{ks}^{*}c_{sl}\left(c_{ks}c_{sl}^{*}-c_{sk}c_{ls}^{*}\right).$$

So, the Gramian expression shows that both EPR coefficient functions can be linearly independent in general, therefore a different result of the action of the selector over the EPR function has to be expected.

7.2. Symmetric–antisymmetric wavefunction linear combination

In order to have even more information on the behavior of the selectors, suppose one constructs the mixed wavefunction:

$$\Phi\left(1,2\right)=N\left(\Psi\left(1,2\right)\pm\Psi\left(2,1\right)\right),$$

where the constant N is a normalization constant. The action of the selector on system S(1) will be:

$$A_s\left(1\right)\left[\Phi\right]=N(A_s\left(1\right)\left[\Psi\left(1,2\right)\right]\pm A_s\left(1\right)\left[\Psi\left(2,1\right)\right]).$$

One already has computed the first term of the expression above, with the result:

$$A_s\left(1\right)\left[\Psi\left(1,2\right)\right]=\alpha_s f_s\left(2\right)a_s\left(1\right)$$

and the second term can be easily computed too, as:

$$\begin{aligned}A_s\left(1\right)\left[\Psi\left(2,1\right)\right]&=\sum_{k,l}c_{lk}A_s\left(1\right)\left[a_l\left(1\right)\right]a_k\left(2\right)\\&=\sum_{k,l}c_{lk}\alpha_s a_s\left(1\right)\delta_{sl}a_k\left(2\right)\\&=\left(\sum_l c_{sk}a_k\left(2\right)\right)\alpha_s a_s\left(1\right)\\&=\alpha_s g_s\left(2\right)a_s\left(1\right)\end{aligned}$$

so, the net result becomes:

$$A_s\left(1\right)\left[\Phi\left(1,2\right)\right]=N\alpha_s\left(f_s\left(2\right)\pm g_s\left(2\right)\right)a_s\left(1\right).$$

At the same time, for the same selector acting on system S(2) it will be obtained:

$$A_s\left(2\right)\left[\Phi\left(1,2\right)\right]=N\alpha_s\left(g_s\left(1\right)\pm f_s\left(1\right)\right)a_s\left(2\right).$$

So, in this situation, while employing a linear superposition of the initial function and its counterpart with the coordinates of the EPR systems exchanged, it is found that the result of the selectors action will be equivalent for the symmetric composition, but a change of sign will appear in the antisymmetric combination case. This can be related to the conditions of the equality of the complete Hermitian operator expectation values discussed in the previous section.

7.3. Selector expectation values

Even with this previous finding, it will be interesting to see how quantum mechanical statistics will affect the selector EPR effect.

The expectation value of a given selector, when applied to system S(1) appears to be expressible as:

$$\begin{aligned}\left\langle A_s(1)\right\rangle &= \left\langle\Psi\middle|A_s(1)\middle|\Psi\right\rangle = \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\left\langle a_l \mid a_q\right\rangle\left\langle a_k\middle|A_s\middle|a_p\right\rangle \\ &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\delta_{lq}\alpha_s\left\langle a_k \mid a_s\right\rangle\left\langle a_s \mid a_p\right\rangle \\ &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\delta_{lq}\alpha_s\delta_{ks}\delta_{sp} \\ &= \sum_{l} c_{ls}^{*}c_{ls}\alpha_s = \left(\sum_{l}|c_{ls}|^2\right)\alpha_s.\end{aligned}$$

In case that the same selector is used over the system S(2), then one obtains the following expectation value:

$$\begin{aligned}\langle A_s(2)\rangle &= \langle\Psi|\,A_s(2)\,|\Psi\rangle = \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\,\langle a_l|\,A_s\,|a_q\rangle\langle a_k \mid a_p\rangle \\ &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\alpha_s\,\langle a_l|a_s\rangle\,\langle a_s|a_q\rangle\,\delta_{kp} \\ &= \sum_{k,l}\sum_{p,q} c_{lk}^{*}c_{qp}\alpha_s\delta_{ls}\delta_{sq}\delta_{kp} \\ &= \sum_{k} c_{sk}^{*}c_{sk}\alpha_s = \left(\sum_{k}|c_{sk}|^2\right)\alpha_s.\end{aligned}$$

Both expectation value expressions cannot further simplify and become identical, unless one assumes that in the two situations the sum of the squared EPR coefficient modules has the same constant value.

Thus, in principle, a similar problem as in the statistical measure of the whole operator appears, when one employs selectors instead. Therefore, one can consider this result as the equivalent appearance, in a unique EPR wavefunction framework, of the EPR effect [2]. It can be consequently stated that: the expectation value of the same selector, when measured in both EPR systems, *apparently can provide different results*.

The analysis of the previous situation, as resumed in equation (15), can be used here to obtain the same weight for both expectation values and, thus, the same result for both state selector measures.

In order that this could be accomplished, it is necessary the following equality hold:

$$\forall s : \sum_k |c_{sk}|^2 = \sum_k |c_{ks}|^2. \tag{23}$$

This is equivalent to consider that the two EPR coefficient functions, being constructed over an orthonormalized basis set, have the same Euclidian norm value (see equations (21) and (22)). That is:

$$\forall s : \left\langle f_s \mid f_s \right\rangle = \left\langle g_s \mid g_s \right\rangle.$$

Equation (23) above will be true when the EPR coefficient matrix becomes Hermitian or Skewhermitian, as has been discussed before, when the complete Hermitian operator was used to find the expectation value.

One should consider also the constraint of dealing with a matrix, simply computed as the matrix **D**, defined in equation (9), previously supposing nothing on his construction structure, but taking into account the additional property to have the sums of row elements equal to the sums of column elements. This property holds for symmetric matrices like the matrix **D**. It must be noted here that one cannot further manipulate the matrix **D** as to become a double stochastic matrix or even a simpler column or row stochastic one [8], because this will destroy its symmetry and normalization conditions (9), that is: $\langle \mathbf{D} \rangle = 1$.

8. Conclusions

One can deduce from the results of this paper that, Hermitian and Skewhermitian EPR coefficient matrices, a situation associated to bosons and fermions, respectively, override the EPR effect in statistical expectation values of a Hermitian operator acting on both EPR systems. However, the case of a selector operator is more complicated.

The EPR effect persists in the expectation values of the selector case, except if the EPR coefficient matrix is Hermitian or Skewhermitian, as in the full operator study. Then, the structure of the EPR coefficient transformed matrix becomes a symmetric matrix and the selector expectation values become equal on both EPR systems.

In any case, the persistence of the EPR effect, when considering the images of the selectors over the EPR wavefunction over both systems seems as solid as in the original EPR two-operator case.

One can conclude, from the point of view of quantum statistical measures, that the EPR effect can be overridden if some specific, physically reasonable, constraints are present into the EPR coefficient matrix and thus into the EPR wavefunction.

Nevertheless, it seems evident that similar constraints as in the statistical case can be found, when the selector wavefunction images are considered. Apparently two different results could be obtained when considering the reaction of EPR system S(1) or S(2) in front of the selector action over the EPR wavefunction. However, a linear symmetric or antisymmetric mixture of the original EPR function with its counterpart having the EPR systems coordinates exchanged provide a selector action on both EPR systems with equivalent results.

Acknowledgments

The author expresses his acknowledgment to the Ministerio de Ciencia y Tecnología for the grant: BQU2003-07420-C05-01; which has partially sponsored this work and for a Salvador de Madariaga fellowship reference: PR2004-0547, which has made possible his stage at the Ghent University. The author wishes to thank Professor Andreas Savin of the Université Pierre et Marie Curie, Paris, for the enlightening comments and the deep discussion of the problems met in an early stage of the construction of the present work. The warm hospitality of Professor Patrick Bultinck of the Ghent University is heartily acknowledged.

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